Universe in a Grain of Salt

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Our universe behaves like a crystal and the matters are just excitations from the ground state. In this paper, we learn that special relativity is nothing special, it is equivalent to an elastic theory of lattice waves and dislocations. In the long wavelength limit, the ether is completely invisible and what we see are excitation modes of ether. Since we cannot see the ether though it may exist, then there is still no preferred frame of reference and hence it does not contradict the Lorentz invariance. However, when the energy scale is around the Planck scale, it is possible for the background material, ether, to show up and provides an absolute frame of reference and hence breaks Lorentz invariance. Theory of string-net condensation consolidates the picture of ether. Gauge bosons and fermions are shown to be the results of the motions of the invisible string-nets on a cubic lattice.

I. INTRODUCTION

Physicists in the 19th century finally abandoned the idea of ether after the experiment by Michaelson and Moseley; its strange mechanical properties render it untenable even in a conceptual way. However, what was disproved is not the elastic idea about vacuum but treating particles as alien objects to the ether [1]. After several decades, dislocation, a very special kind of defect in solid, was discovered. The interesting thing is that dislocations behave like particles with charge and mass and a form of special relativity emerges in an elastic medium predicting that propagation speed of dislocation is bounded by speed of sound and that the stress field is Lorentz contracted [1]. The question is raised: whether or not can we treat the vacuum as an elastic medium, the ether, which implies Lorentz invariance?

If the answer is yes, then we should consider the next question: can Lorentz invariance be broken at very high energy scale? What the elastic theory of crystals based on is actually the long wavelength limit of a general phonon dispersion relation.[4] In the vicinity of the origin, the dispersion relation resembles the linear photon (massless particles) dispersion relation. It provoked us to think about the breaking of Lorentz invariance when it is far from the origin, namely extremely short wavelength, the Planck scale, many people believe [5–8].

The above discussion is classical or parallel to Einstein's approach. Are there quantum mechanical theories which support the idea of ether and emergence of elementary particles and fundamental laws? Yes, elementary particles and there interactions can be viewed as emergent from the effective theory of bosonic spin on a cubic lattice [9].

This paper surveys the modern ideas of ether and follows the concept of emergence. I will first discuss the elastic theory of special relativity and then move to the breaking of Lorentz invariance from a phonon point of view and finally introduce the idea of string-net which gives a closer look at the ether.

II. THE ELASTIC SPECIAL RELATIVITY

A. Dislocations

Dislocation is a kind of line defect found in crystals usually formed in the growth process but it can be easily visualized via Volterra construction. First, an imaginary cut is made into the crystal and it terminates inside the crystal, at an arbitrary line called dislocation line. Then the separated two parts are displaced relative to each other and depending on the displacement, dislocations are put into two major categories: edge and screw dislocations. If the displacement is perpendicular to the dislocation line, we have an edge dislocation; if the displacement is along the line, then it is a screw dislocation (Figure 1 and 2). General cases may involve curved dislocation lines, then Burgers circuits and vectors are indispensable (Figure 3). Burgers circuit is strongly reminiscent of 'parallel transport' in Riemannian geometry, actually, the dislocation corresponds to torsion while another kind of defect called disclination corresponds to curvature. Disclination is the case where the cut is made and then an angle is inserted or removed, finally the cut is glued together. It is the same as making a cone (a saddle surface) which has positive (negative) curvature.

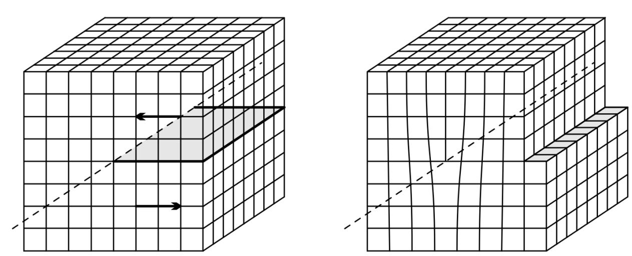

FIG. 1. Volterra construction of an edge dislocation

We can also characterized the atom displacements by a vector field $\boldsymbol{u}(\boldsymbol{r})$ and restrict it in the Wigner-Seitz

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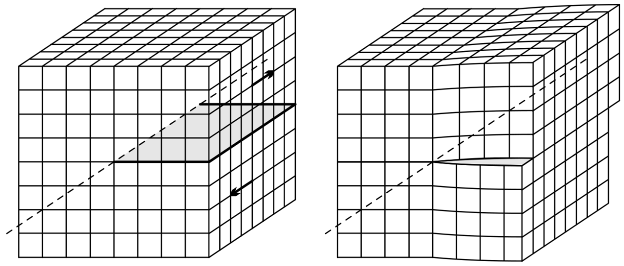

FIG. 2. Volterra construction of a screw dislocation.

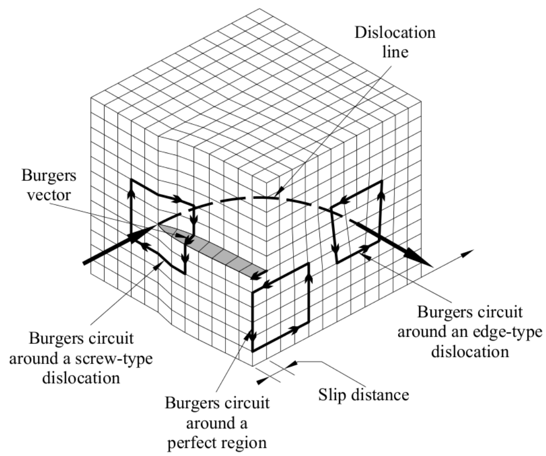

FIG. 3. Burgers circuits and Burgers vectors.

cell (periodic boundary conditions implied). When the Burgers circuits are far from the dislocation line, the displacements are small and can be considered continuous, moreover, the unwanted jumps can be removed by identifying opposite sides of Wigner-Seitz cell, namely transforming the n-dimensional cell into a n-dimensional torus. In this case, the Burgers vector is a loop integral along the Burgers circuit (Γ)

$$\boldsymbol{b} = \oint_{\Gamma} \frac{d\boldsymbol{u}}{ds} ds \tag{II.1}$$

Deformation of the circuit does not change the winding numbers of path traversed on the torus, in the parameter space ($\boldsymbol{u}$-space), therefore, dislocations are also called topological defects and a dislocation can be completely characterized by a set of n winding numbers which is equivalent to n components of a Burgers vector.

B. Emergence of special relativity

Frank [1] found out that the limiting speed of the motion of dislocations turned out to be speed of sound similar to a particle with nonvanishing mass in special relativity. He also noticed the stress field would experience similar contraction as that of a relativistic electron.

Frank considered a displacement field of a screw dislocation

$$u_x = 0 = u_y, \quad u_z = \frac{b}{2\pi} \arctan \frac{y}{x} \tag{II.2}$$

where we have

$$\nabla \cdot \boldsymbol{u} = \partial_z u_z = 0 \tag{II.3}$$

Then the Navier equation that governs the displacement field

$$\rho \ddot{\boldsymbol{u}} = (\lambda + \mu) \nabla (\nabla \cdot \boldsymbol{u}) + \mu \nabla^2 \boldsymbol{u} \tag{II.4}$$

reduces to wave equation,

$$\rho \ddot{\boldsymbol{u}} - \mu \nabla^2 \boldsymbol{u} = 0 \tag{II.5}$$

If the dislocation is propagating in the x-direction, we make substitution

$$x' = x(t) - vt = \text{const. w/r } t, \quad \partial_t = -v \partial_{x'} \tag{II.6}$$

the wave equation becomes

$$\left\{ (\partial_y^2 + \partial_z^2) + (1 - v^2 \rho/\mu) \partial_{x'}^2 \right\} \boldsymbol{u} \equiv (\partial_{x''}^2 + \partial_y^2 + \partial_z^2) \boldsymbol{u} = 0 \tag{II.7}$$

where we have the 'Lorentz transformation,'

$$x'' = \frac{x'}{\sqrt{1 - v^2 \rho/\mu}} = \frac{x - vt}{\sqrt{1 - v^2/c^2}} \tag{II.8}$$

with $c = \sqrt{\mu/\rho}$, the speed of sound.

It is also shown by Frank that the energy transforms as

$$E = \frac{E_0}{\sqrt{1 - v^2/c^2}} \tag{II.9}$$

as well as time dilation by Gunther [3].

The above results holds if $\nabla \cdot \boldsymbol{u} = 0$, which corresponds to an incompressible medium. The idea of incompressible medium may sound problematic when applied to the vacuum since it had been proved by Michaelson-Moseley experiment that ether does not exist. However, what they disproved is not the existence of such a medium but the weird argument that matters are alien to ether and they perform frictionless motion even at very high speed. Modifying the argument by taking the the particles as excitations of ether will smooth out the wrinkles, we will come to the issue in later sections.

C. Emergence of Maxwell equations

1. *Sourceless Maxwell equations*

Using the vector identity,

$$\nabla \times \nabla \times \boldsymbol{a} = \nabla (\nabla \cdot \boldsymbol{a}) - \nabla^2 \boldsymbol{a} \tag{II.10}$$

Eq (II.4) can be rewritten in equivalent form,

$$\rho\ddot{\boldsymbol{u}} = (\lambda + 2\mu)\nabla(\nabla\cdot\boldsymbol{u}) - \mu\nabla\times\nabla\times\boldsymbol{u} \quad \text{(II.11)}$$

Now we apply the incompressible condition $\nabla\cdot\boldsymbol{u} = 0$ and identify electric field $\boldsymbol{E} = \nabla\times\boldsymbol{u}$ and magnetic field $\boldsymbol{B} = \dot{\boldsymbol{u}}$ according to MacCullagh [2], we have ($c = 1$)

$$\dot{\boldsymbol{B}} = -\nabla\times\boldsymbol{E}, \quad \nabla\cdot\boldsymbol{B} = \partial_t\nabla\cdot\boldsymbol{u} = 0 \quad \text{(II.12)}$$

and the other pair,

$$\dot{\boldsymbol{E}} = \nabla\times\dot{\boldsymbol{u}} = \nabla\times\boldsymbol{B}, \quad \nabla\cdot\boldsymbol{E} = \nabla\cdot\nabla\times\boldsymbol{u} = 0 \quad \text{(II.13)}$$

The elastic equation 'contains' Maxwell's four equations in vacuum.

2. *Maxwell equations with source*

Dislocations behave like quantum mechanical particles for the following reasons. First, all dislocations with the same Burgers vector are the same, they are indistinguishable 'particles.' Second, dislocations with opposite Burgers vectors will annihilate (conservation of Burgers vector: $\boldsymbol{b} = \boldsymbol{b}_1 + \boldsymbol{b}_2$) and the energy stored will be released through phonons. It is similar to electron-positron annihilation. Third, the opposite is also possible, i.e. pair creation, during propagation.

The sources can be taken into the model by identifying topological defects as electric charges but we need to extend the linear theory to nonlinear theory. We first introduce a deformation tensor F and make polar decomposition $\mathsf{F} = \mathsf{QR}$ where Q is an orthogonal matrix, an element in SO(3). The rotational part is identified with the electric field. Then the charges, or topological defects, can be found from the homotopy groups of SO(3).

III. BREAKING OF LORENTZ INVARIANCE

A. Dispersion relation

Dispersion relation is the energy momentum relation. In Newtonian mechanics, kinetic energy is quadratic in momentum but latter it proves to be just a quadratic approximation to a general dispersion relation of massive object described in special relativity. However, in crystals, lattice waves' dispersion relations are quite different from the ones in relativity. Then, is the dispersion relation derived in special relativity general enough even at extremely high energy scale? There is no law of nature that guarantees the Lorentz invariance and this symmetry spontaneously breaks in several theories.

For a 1-dimensional monatomic chain, the dispersion relation reads,

$$\omega(q) = \sqrt{\frac{2K(1-\cos qa)}{M}} = 2\sqrt{\frac{K}{M}}\left|\sin\frac{qa}{2}\right| \quad \text{(III.1)}$$

where ω, q, a, K and M are frequency, wave vector, lattice constant, force constant (quadratic coefficient) and mass of atom. In the long wavelength limit, $q \ll 1/a$, the dispersion relation reduce to

$$\omega(q) = \sqrt{\frac{K}{M}}|q|a \equiv c|q| \quad \text{(III.2)}$$

where $c = a\sqrt{K/M}$ is the speed of sound (speed of elastic wave). The two relations are plotted in Figure 4.

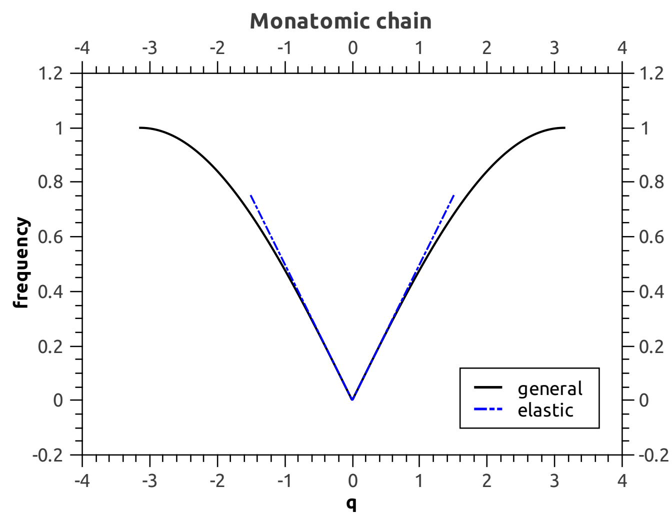

FIG. 4. (Color online) Dispersion relation of a monatomic chain where near the origin, it can be approximated by a linear relation.

As pointed out in the previous section, speed of sound serves as 'speed of light' in a crystal and we may argue that Lorentz invariance only holds in the long wavelength limit. The lattice constant of vacuum is believed by many theorists to be of the order of Plank length or an energy equivalent to 10^{19} GeV

For a simple cubic lattice, there are three acoustic branches in total: two transverse modes and one longitudinal mode. This is similar to three polarization states of photons. The two transverse modes are degenerate in some directions and split in other directions but the speed of transverse elastic wave is common for two modes. Speeds of elastic waves are,

$$c_L = \sqrt{\frac{\lambda + 2\mu}{\rho}}, \quad c_T = \sqrt{\frac{\mu}{\rho}} \quad \text{(III.3)}$$

in general $c_L > c_T$. This is different from the fact that photons of different polarizations have unique speed. Or the contrary, we may detect differences in speed of photons of three polarization states, the small differences can be explained if ρ is extremely large, or vacuum is an extremely dense material, then the factors μ, λ can be neglected when compared to ρ.

The cosmological red shift can be easily explained if we consider the vacuum as lattice of lattice constant $a(t)$.

Now that we increase the constant $a(t)$ adiabatically, the quantity $\dot{a}/a$ is just the Hubble's constant.

B. Consequences of anharmonicity

The independent collective excitations, the phonons, stems from the harmonic approximation to the crystal potential. In this regime, the Hamiltonian is diagonalized in terms of creation and annihilation operators,

$$\mathcal{H} = \sum_{\boldsymbol{q},\lambda} \hbar\omega_\lambda(\boldsymbol{q}) \left(a_\lambda^\dagger(\boldsymbol{q}) a_\lambda(\boldsymbol{q}) + \frac{1}{2} \right) \qquad \text{(III.4)}$$

when higher order terms in the potential expansion are involved, terms like $a^\dagger aa$ or $a^\dagger a^\dagger aa$ will be present in the Hamiltonian [4]. It this case, phonons may scatter each other and decay. Due to the concave shape of dispersion relation, phonons from the same branch cannot decay, since energy and momentum cannot be conserved simultaneously.

If photons have finite life, then it has nonvanishing mass and if photons can decay then there will be selection rules concerning there polarizations. We can test the anharmonicity effect for photons, if there is, by the above analysis.

IV. PHOTONS AND ELECTRONS

A. Toy universe in a superfluid

In a superfluid, there are two kinds of excitations, the phonons and rotons. If the background bosons are invisible, just like ether, what the inhabitants in the toy universe see are the above two kinds of 'elementary' particles. Photons in our universe induce electromagnetic interactions, similarly, phonons are also responsible for interactions in the toy universe [9].

Charges of such interactions can be defined by the flux of the background bosons: positive phonon charge is a source of background boson and negative charge is a drain thereof. The interactions between the charges is found to be similar to Coulomb interaction with a potential q_1q_2/r.

However, the total number of background bosons is conserved, there will not be net charges in the toy universe. The flow patterns of background bosons about the roton can be viewed as a flow of phonon charges in multipole expansion. Therefore, the rotons interact through dipolar interaction $(1/r^4)$ by exchanging massless phonons.

B. QED and QCD on a cubic lattice

Can Coulomb interaction $(1/r^2)$ emerge naturally from ether? The string-net condensation provides an answer to the origin of light and fermions and unifies gauge interactions and Fermi statistics according to Wen [9]. The U(1) gauge field and anti-commuting fermion fields were manually introduced in previous models, thanks to the advance of local bosonic theories, gauge bosons and fermions can be seen as emergent particles. Furthermore, one may construct ugly bosonic spin models on a cubic lattice the low-energy effective theory of which is the beautiful QED and QCD with emergent photons, electrons, quarks and gluons. The Coulomb interaction $(1/r^2)$ can, indeed, emerge from a lattice theory.

According to string-net theory, light is the fluctuation of condensed string-nets and fermions are ends of condensed strings, the collective motions of the string-nets form photons and fermions.

V. CONCLUSIONS

In this paper, we learn that special relativity is nothing special, it is equivalent to an elastic theory of lattice waves and dislocations. In the long wavelength limit, the ether is completely invisible and what we see are excitation modes of ether. It is similar to the toy universe where the background bosons are invisible and excitations, phonons and rotons are considered 'elementary particles.' Since we cannot see the ether though it may exist, then there is still no preferred frame of reference and hence it does not contradict the Lorentz invariance.

However, when the energy scale is around the Planck scale, it is possible for the background material, ether, to show up and provides an absolute frame of reference and hence breaks Lorentz invariance. It is equivalent to test the validity of dispersion relations given by special relativity and this test may serve as the evidence that supports the existence of ether.

Our universe behaves like a crystal and the matters are just excitations from the ground state. Theory of string-net condensation clarifies this picture. Gauge bosons and fermions are results of the motions of the invisible string-nets on a cubic lattice.

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